Economic Growth & Development: Part 4
Vertical Innovation Models

By Kiminori Matsuyama

Updated on 2011-04-14, 11:01:54 AM

Introduction

- In the previous models, R&D develops products that are new, i.e., imperfect substitutes of the existing products. The economy grows through "expanding variety."

- We now study models where a newly developed product is a perfect substitute of the existing product. The new product is a "better or improved" version of the old one, which it is going to replace. Products are "vertically differentiated." The economy grows through "quality improvement."

- Each product has its own "life-cycle." It first replaces older vintages, but it will eventually be replaced by a new product in the future. Temporary monopoly power.

- Subtle welfare implication. On one hand, innovators do not value the monopoly profit earned by the producer of the old vintage that its successful innovation will destroy; this works in the direction of over-investment. On the other hand, they also know that the return to innovation is only temporary; this works in the direction of under-investment.

- These models may also be interpreted as models of process innovations. Each innovation comes up with a new way of producing the goods at a reduced cost.

Lab-Equipment Version: (Acemoglu Ch.14.1)

Final Good Production:

$$Y(t) = \frac{1}{1-\beta} \bar{X}(t)^{1-\beta} L^{\beta}, \qquad \text{where } \bar{X}(t) \equiv \left[\int_0^1 \left[\sum_{q \in I(\nu,t)} q^{\zeta_1} x(\nu, t | q) \right]^{1-\frac{1}{\sigma}} d\nu \right]^{\frac{\sigma}{\sigma-1}}$$

- A continuum of industries, $\nu \in [0,1]$, each producing a particular line of intermediates.
- $I(\nu,t)$: The range of quality available for product line ν at time t.
- q: the quality index of each product
- $x(\nu, t | q)$; the units of the product used of quality q in product line ν at time t.

Within each product line, products of different quality are perfect substitutes. It turns out that, in equilibrium, only the product of highest quality available, denoted by $q(\nu, t)$, is used at each moment. We also assume $\beta = 1/\sigma$ and index quality such that $\zeta_1(1 - \beta) = 1$.

$$\Rightarrow Y(t) = \frac{1}{1-\beta} \left[\int_0^1 q(\nu, t) \left[x(\nu, t | q(\nu, t)) \right]^{1-\beta} d\nu \right] L^{\beta}.$$

"Quality Ladder": $q(v,t) = \lambda^{n(v,t)} q(v,0)$.

- $n(v,t)$; # of successful innovations between 0 and t in product line v, a random variable.
- Within each product line, a new innovation improves the quality by factor of $\lambda > 1$.

R&D and Production Technologies for Intermediates:

- R&D is cumulative in the sense that it builds on the experiences of previous R&D.
- Only with $q(v,t)$ currently available, it is feasible to invent quality, $\lambda q(v,t)$.
- Investing $Z(v,t)$ units of the final good generate a flow rate of success (Poisson arrival rate) equal to $\dfrac{\eta Z(v,t)}{[q(v,t)]^{\zeta_2}}$.
- Only new entrants conduct such R&D, not by the incumbent, which currently produces $q(v,t)$. The incumbent has weaker incentives, because it would replace its own product, thus destroying the profits that they are currently making. ***Arrow's replacement effect.***
- Once invented, one unit of product of quality q can be produced with $\psi(q)^{\zeta_3}$ units of the final good.
- Assume $\zeta_2 = \zeta_3 = 1$. (We need some restriction on ζ_1, ζ_2, and ζ_3 to ensure the BGP; $\zeta_1(1-\beta) = \zeta_2 = \zeta_3 = 1$ is one such restriction, but not the only one.)

Demand for an intermediate: $x(\nu,t) = L\left(\dfrac{q(\nu,t)}{p^x(\nu,t)}\right)^{1/\beta}$.

Monopoly pricing: Each quality leader has quality advantage of $(\lambda)^{\zeta_1} = (\lambda)^{1/(1-\beta)}$ and cost disadvantage of $(\lambda)^{\zeta_3} = \lambda$ over the previous leader. In order to replace the previous leader, its price must satisfy $p^x(\nu,t)/\lambda^{1/(1-\beta)} < \psi/\lambda \Rightarrow p^x(\nu,t) < \lambda^{\beta/(1-\beta)}\psi$. This constraint is not binding if λ is sufficiently large (the drastic innovation case) such that,

$$\lambda > \left(\frac{1}{1-\beta}\right)^{\frac{1-\beta}{\beta}}.$$

In this case, the leader sets $p^x(\nu,t)(1-\beta) = \psi q(\nu,t)$. Normalize $\psi = 1-\beta$. Then,

$$p^x(\nu,t) = q(\nu,t),\ x(\nu,t) = L, \text{ and } \pi(\nu,t) = \beta q(\nu,t)L \text{ for all } \nu \ \& \ t$$

as long as it remains the quality leader.

$$\Rightarrow Y(t) = \frac{Q(t)}{1-\beta}L,\quad X(t) = (1-\beta)Q(t)L,\ \&\ w(t) = \frac{\beta Q(t)}{1-\beta}$$

where $Q(t) \equiv \int_0^1 q(\nu,t)d\nu$ is the average quality across sectors, which is the engine of growth.

Value of a Quality Leader: Even if each product is forever protected by patent, its value is destroyed when it is replaced by innovation of a better product.

$$r(t)V(v,t|q) = \dot{V}(v,t|q) + \pi(v,t|q) - z(v,t|q)V(v,t|q)$$

where $z(v,t|q) = \eta Z(v,t|q)/q$ is the flow rate at which a successful innovation takes place in v at t.

R&D (Innovation): Free entry

$$\eta V(v,t|q) \leq q/\lambda \quad \text{and } \eta V(v,t|q) = q/\lambda \qquad \text{if } z(v,t|q) > 0.$$

Note: Innovators make zero profit. This means that, when they innovate across sectors, they are indifferent about how much they invest in each sector.

Evolution of *Q*(*t*): suppose $z(v,t|q) = z(t)$ for all v at t. In an interval of time, δt,

- $z(t)\delta t$ sectors experience one innovation, which will increase their quality by λ.
- The measure of sectors experiencing more than one innovation is $o(\delta t)$.

$$\Rightarrow Q(t+\delta t) = \lambda Q(t)z(t)\delta t + Q(t)(1-z(t)\delta t)) + o(\delta t) \Rightarrow \dot{Q}(t) = (\lambda - 1)z(t)Q(t).$$

Since $z(v,t|q) = z(t) \Rightarrow z(t)q(v,t) = \eta Z(v,t|q) \Rightarrow z(t)Q(t) = \eta Z(t)$,

$$\dot{Q}(t) = (\lambda - 1)\eta Z(t).$$

No aggregate fluctuation because there are many (a continuum of) sectors.

Characterizing BGP: look for BGP along which

- $r(t) = r^*$ such that $\dot{Q}/Q = \dot{Y}/Y = \dot{C}/C = (r^* - \rho)/\theta > 0$;
- From the aggregate resource constraint, $Y(t) = C(t) + X(t) + Z(t)$,

$$\frac{L}{1-\beta} = \frac{C(t)}{Q(t)} + (1-\beta)L + \frac{Z(t)}{Q(t)} = \frac{C(t)}{Q(t)} + (1-\beta)L + \frac{z(t)}{\eta} \Rightarrow z(t) = z^* > 0.$$

Free Entry: $\Rightarrow V(v,t|q) = \dfrac{q}{\eta\lambda} \equiv V(q)$

Valuation of a Firm: $\Rightarrow V(q) = \dfrac{\pi(q)}{r^*+z^*} = \dfrac{\beta q L}{r^*+z^*}$

Combining these, $r^*+z^* = \lambda\eta\beta L$

$$\Rightarrow \frac{r^*-\rho}{\theta} = g^* = \frac{\dot{Q}}{Q} = (\lambda-1)z^* = (\lambda-1)(\lambda\eta\beta L - r^*)$$

$$\Rightarrow g^* = \frac{\lambda\eta\beta L - \rho}{\theta + (\lambda-1)^{-1}} > 0.$$

From $\dfrac{L}{1-\beta} = \dfrac{C(t)}{Q(t)} + (1-\beta)L + \dfrac{z^*}{\eta}$,

$$C(t) = \left[\frac{L}{1-\beta} - (1-\beta)L - \frac{z^*}{\eta}\right]Q(t).$$

For the existence, we need $\lambda\eta\beta L > \rho > (1-\theta)\dfrac{\lambda\eta\beta L - \rho}{\theta + (\lambda-1)^{-1}}$.

Notes:

- Again, the scale, efficiency, and taste parameters have the growth effects.
- Starting from any $Q(0) > 0$, there is an equilibrium path along which the economy grows at the constant rate, *g**. But, I am not convinced that this is the only equilibrium path (although Acemoglu asserts that it is).
- The optimal growth is also a balanced growth path. Unlike the horizontal innovation models, the optimal growth rate can be higher or lower than the growth rate in the equilibrium balanced growth path.

Tax Policy on R&D spending:

By discouraging R&D, this increases the value of an incumbent as $V(q) = \dfrac{(1+\tau)q}{\eta\lambda}$.

Since $V(q) = \dfrac{\beta qL}{r^* + z^*}$,

$$\Rightarrow r^* + z^* = \frac{\lambda\eta\beta L}{1+\tau} \Rightarrow \frac{r^* - \rho}{\theta} = g^* = (\lambda - 1)z^* = (\lambda - 1)\left(\frac{\lambda\eta\beta L}{1+\tau} - r^*\right)$$

$$\Rightarrow g^* = \frac{1}{\theta + (\lambda - 1)^{-1}}\left(\frac{\lambda\eta\beta L}{1+\tau} - \rho\right).$$

Hence, it reduces the growth rate.

Aghion-Howitt Model: (Acemoglu Ch.14.2)

Innovation by Both Incumbents & Entrants: (Acemoglu Ch.14.3)

So far, R&D is done only by entrants. This model allows R&D both by incumbents and entrants. The lab-equipment version (Ch.14.1) is modified as follows:

- A small incremental improvement ("tinkering") can be done only by incumbents.
- The parameters are also changed so that, incumbents grow in size as they improve the quality of their products, to obtain the firm size dynamics.
- A drastic innovation is done only by entrants in equilibrium. (Again, incumbents have no incentives to do so in equilibrium due to the Arrow's replacement effect).

Final Goods Production: Index quality such that $\zeta_1(1-\beta) = \beta$.

$$\Rightarrow Y(t) = \frac{1}{1-\beta}\left[\int_0^1 q(\nu,t)^{\beta}\left[x(\nu,t|q(\nu,t))\right]^{1-\beta} d\nu\right]L^{\beta}$$

Intermediate Inputs Production: Once invented, one unit of product of quality q can be produced with $\psi(q)^{\zeta_3} = (1-\beta)(q)^{\zeta_3}$ units of the final good, where $\zeta_3 = 0$; i.e., it is equal to $\psi = 1-\beta$, independent of q.

Small quality improvement (tinkering): R&D available only to incumbents:
"Quality Ladder": $q(v,t) = \lambda^{n(v,t)} q(v,s)$, where $\lambda > 1$

- $n(v,t)$; # of successful improvement between s and $t > s$ in product line v, a random variable.
- s is the date at which this incumbent took over this product line, by making a drastic innovation.
- Tinkering upgrades the current quality level, $q(v,t)$, to the next level, $\lambda q(v,t)$.
- Investing $z(v,t)q(v,t)$ units of the final good by the incumbent generates a flow success rate (Poisson arrival rate) equal to $\dfrac{\phi z(v,t)q(v,t)}{\left[q(v,t)\right]^{\zeta_2}} = \phi z(v,t)$, with $\zeta_2 = 1$.

Drastic innovation (Creative destruction): R&D pursued only by entrants:

- For the current quality $q(v,t)$, a successful drastic innovation leads to $\kappa q(v,t)$, $\kappa > \lambda$.
- Each unit of the final good invested by an entrant in R&D generates a flow success rate of $\eta(\hat{z}(v,t))/q(v,t)$, where $\hat{z}(v,t)$ is the total R&D spending by all entrants, divided by $q(v,t)$, so that the flow success rate is $\eta(\hat{z}(v,t))\hat{z}(v,t)$.
- $\eta(z)$ is strictly decreasing. This captures external diminishing returns, which each entrant takes as given. The negative externalities are mild so that $z\eta(z)$ is strictly increasing. Assume $\lim_{z\to\infty} \eta(z) = 0$; $\lim_{z\to 0} \eta(z) = \infty$, to ensure the interior solution.

Demand for an intermediate: $x(\nu,t) = q(\nu,t)L\left(p^x(\nu,t)\right)^{-1/\beta}$.

Monopoly pricing: Each incumbent has at least quality advantage of $(\kappa)^{\zeta_1} = (\kappa)^{\beta/(1-\beta)}$ over the previous leader, but no cost disadvantage, because of $(\kappa)^{\zeta_3} = 1$. Hence, the leader must set its price such that $p^x(\nu,t) < (\kappa)^{\beta/(1-\beta)}\psi$.

Assume that the innovation by entrants is drastic enough that

$$\kappa > \left(\frac{1}{1-\beta}\right)^{\frac{1-\beta}{\beta}}.$$

Then, the leader sets its monopoly price unconstrained, $p^x(\nu,t) = \psi/(1-\beta) = 1$.

$\Rightarrow x(\nu,t) = q(\nu,t)L$, and $\pi(\nu,t) = \beta q(\nu,t)L$ for all ν & t

$\Rightarrow Y(t) = \dfrac{L}{1-\beta}Q(t)$, $X(t) = (1-\beta)Q(t)L$, and $w(t) = \dfrac{\beta}{1-\beta}Q(t)$

where $Q(t) \equiv \int_0^1 q(\nu,t)d\nu$ is again the average quality across sectors.

Value of an Incumbent producing *q*: Keep the notation simple by $V(v,t|q) = V(q)$;

$$r(t)V(q) = \dot{V}(q) + \pi(q) + \max_{z\geq 0}\{(\phi z)(V(\lambda q) - V(q)) - zq\} - \eta(\hat{z}(q))\hat{z}(q)V(q)$$

R&D by Entrants (Creative Destruction): $\eta(\hat{z}(v,t|q))V(v,t|\kappa q) = q$ with $\hat{z}(v,t|q) > 0$.

There is always some R&D by entrants, since $\lim_{z\to 0}\eta(z) = \infty$.

R&D by Incumbents (Tinkering):

$$\phi(V(v,t|\lambda q) - V(v,t|q)) \leq q; \quad \phi(V(v,t|\lambda q) - V(v,t|q)) = q \qquad \text{if } z(v,t|q) > 0.$$

Evolution of *Q*(*t*): suppose $z(v,t|q) = z(t)$ and $\hat{z}(v,t|q) = \hat{z}(t)$ for all *v*, *q*, and *t*. Then,

$$Q(t+\delta t) = (\lambda\phi z(t)\delta t Q(t) + \kappa\eta(\hat{z}(t))\hat{z}(t)\delta t)Q(t) + (1 - \phi z(t)\delta t - \kappa\eta(\hat{z}(t))\hat{z}(t)\delta t)Q(t) + o(\delta t)$$

$$\Rightarrow \dot{Q}(t)/Q(t) = (\lambda - 1)\phi z(t) + (\kappa - 1)\eta(\hat{z}(t))\hat{z}(t).$$

Balanced Growth Path: Let us look for the BGP where

- $r(t) = r^*$ such that $\dot{C}/C = (r^* - \rho)/\theta \equiv g^* > 0$
- $\hat{z}(v, t|q) = \hat{z} > 0$ and $z(v, t|q) = z > 0$.
- $V(v, t|q) = vq$

Positive R&D by Incumbents $\Rightarrow V(q) = \dfrac{q}{\phi(\lambda - 1)}$.

Valuation of a Firm $\Rightarrow r^*V(q) = \beta qL - \eta(\hat{z})\hat{z}V(q) \Rightarrow V(q) = \dfrac{\beta Lq}{r^* + \eta(\hat{z})\hat{z}}$

$$\Rightarrow r^* = \phi(\lambda - 1)\beta L - \eta(\hat{z})\hat{z};$$

Free Entry by Entrants $\Rightarrow \eta(\hat{z}) = \dfrac{q}{V(\kappa q)} = \dfrac{\phi(\lambda - 1)}{\kappa}$

$$\Rightarrow g^* = \frac{r^* - \rho}{\theta} = \frac{\phi(\lambda - 1)\beta L - \eta(\hat{z})\hat{z} - \rho}{\theta} = \frac{\eta(\hat{z})\kappa\beta L - \eta(\hat{z})\hat{z} - \rho}{\theta}.$$

We also have: $g^* = \dot{Q}/Q = (\lambda - 1)\phi z + (\kappa - 1)\eta(\hat{z})\hat{z}$,

which determines z.

Again, for the existence of this BGP, we must verify:

- Incumbents have an incentive to do R&D.
- $(1-\theta)r^* < \rho < r^*$.

Effects of *λ* & *κ*: From $g^* = \dfrac{\phi(\lambda-1)\beta L - \eta(\hat{z})\hat{z} - \rho}{\theta}$ and $\eta(\hat{z}) = \dfrac{\phi(\lambda-1)}{\kappa}$,

$$\lambda \Uparrow \rightarrow \hat{z} \Downarrow \rightarrow \eta(\hat{z})\hat{z} \Downarrow \rightarrow g^* \Uparrow; \qquad \kappa \Uparrow \rightarrow \hat{z} \Uparrow \rightarrow \eta(\hat{z})\hat{z} \Uparrow \rightarrow g^* \Downarrow.$$

- More creative destruction reduces the growth rate.
- This is because it reduces the incremental innovation by the incumbents:

  $\kappa \Uparrow$, $\eta(\hat{z})\hat{z} \Uparrow$, and $g^* \Downarrow$ ➔ $z \Downarrow$ from $g^* = (\lambda-1)\phi z + (\kappa-1)\eta(\hat{z})\hat{z}$.

Indeed, Acemoglu (Proposition 14.6) show that, while taxation on R&D spending by incumbents are growth-reducing, taxation on R&D spending by entrants are growth-enhancing, in strong contrast with the baseline model of Ch.14.1.
My partial intuition: Encouraging R&D by the entrants discourages R&D by the incumbents, while encouraging R&D by the incumbents will not discourage R&D by the entrants, because they do R&D to become incumbents.

Acemoglu also discusses the model's implication on firm size dynamics.

Step-By-Step Innovation: (Acemoglu Ch.14.4)